Work and Energy Sample Problems

1. A 5 N object slides down a 20° incline which has a coefficient of friction of 0.30. What work is done with gravity? What work is done against friction? What is the net work done? Predict the final velocity of the object if it starts from rest and slides down the 2m long incline.

Ans: +3.42 J; -2.82 J; 0.60 J; 1.53 m/s

2. A 32 kg child is released from rest at the top of a curved water slide, 8.5 m above the level of the pool. How fast is the child moving when he is projected into the pool? Assume that the slide is frictionless.

Ans: 12.91 m/s

3. A 5.2 g ball bearing is fired vertically downward from 18 m with an initial speed of 14 m/s. It buries itself a depth of 21 cm in the sand. What average resistive force does the sand exert on the ball bearing?

Ans: -6.81N

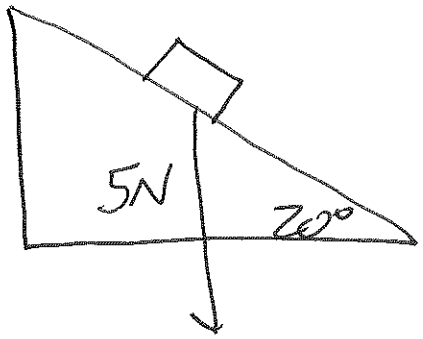
4. A 4.5 kg block is fired up a 32° incline with an initial speed of 5.2 m/s. It travels 1.5 m up the plane, momentarily comes to rest, and then slides down to the bottom. What is the magnitude of the frictional force acting on the block? What is the speed of the block as it returns to the bottom of the incline?

Ans: 17.16 N; 2.04 m/s

5. A child is swinging on a swing. At the swing's maximum height from the ground, the ropes of the swing make an angle of 20° with their point of attachment. The ropes of the swing are 3 m long. What is the child's maximum speed? Assume frictionless conditions.

Ans: 1.88 m/s

(1)

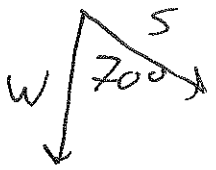


$\mu_K = .3$

a) W_G?

$W = F \times s \times \cos\theta$

$5N \times 2m \cdot \cos 70°$

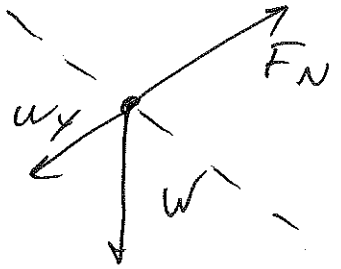


$= 3.42\,J$

b) W by Friction

$W = F_{s_K} \times s \times \cos\theta$

$\mu_K F_N \times s \times \cos\theta$



so $F_N = W_y = mg\cos 20$

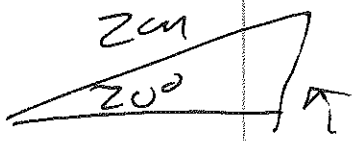
$\mu_S mg\cos 20° \cdot 2m \cdot \cos 180°$

$.3(5N)\cos 20° \cdot 2\cos(180)$

$W_{Friction} = -2.82\,J$

c) Net work $= 3.42\,J + -2.82\,J$

$= 0.6\,J$



d) $W_{NC} = E_F - E_0$

$K_F + U_F - (K_0 + U_0)$

$-2.82\,J = \frac{1}{2}mv_F^2 + 0\,J - (0\,J + mgh_0)$

$-2.82\,J = \frac{1}{2}\left(\frac{5}{9.8}v_F^2\right) - 5N(h_0)$

$h = 2\sin 20°$

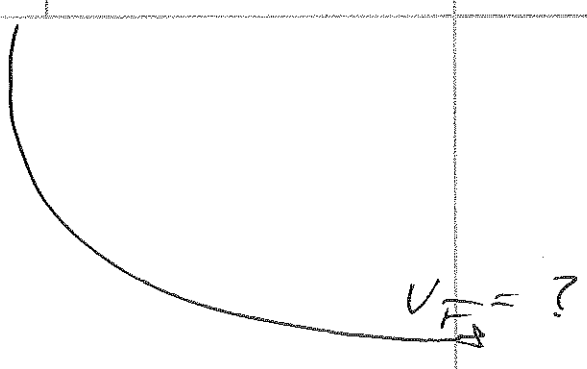
$-2.82\,J = .25v_F^2 - 5 \cdot 2\sin 20°$

$v_F = 1.53\,m/s$

(2) $m = 32\,kg$

$V_0 = 0$

$h_0 = 8.5\,cm$



$E_0 = E_F$

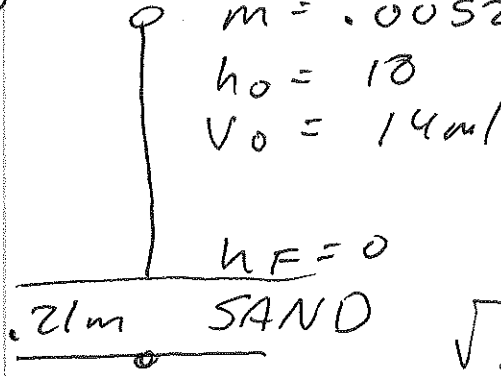
$KE_0 + U_0 = KE_F + U_F$

$0 + mgh_0 = \frac{1}{2}mv_F^2 + 0$

or $v_F = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 8.5\,cm}$

$$\boxed{V_F = 12.91\,m/s}$$

(3)



$E_0 = E_F$

$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_F^2$

$\sqrt{v_0^2 + 2gh_0} = v_F$

$\sqrt{14^2 + 2(9.8)\,18} = v_F = 23.42\,m/s$

So, the SAND does $-W$ on the ball.

$W_{NC} = E_F - E_0$ → Top of SAND

→ Bottom of SAND

$F \times s \times \cos\theta = 0 - \frac{1}{2}mv_0^2$

$F \cdot (.21)(\cos 180^\circ) = 0 - \frac{1}{2} \cdot .0052\,(23.42\,m/s)^2$

$(\cos 180^\circ = -1)$

$$\boxed{F = 6.8\,N}$$

(4) $m = 4.5\ kg$, $1.5\ m$, 32°, h_F

$V_0 = 5.2\ m/s$

a) $W_{NC} = E_F - E_0$

$KE + U$ $\quad$ $KE_0 + U_F$

$F \cdot \$ \cdot \cos(180) = U_F - KE_0$

$F \cdot 1.5 \cdot (-1) = 4.5(9.8)(.8) - \frac{1}{2} 4.5 kg (5.2)^2$

$F \cdot (1.5)(-1) = -25.78\ J$

$\boxed{F = 17.2\ N}$

b) As the block slides down, friction acts in a similar way. $W = -25.8\ J$ so

$W_{NC} = E_F - E_0$

$-25.8\ J = \frac{1}{2} m v_F^2 - mgh_0$

$-25.8\ J + (4.5 kg)(9.8)(.8m) = \frac{1}{2}(4.5) v_F^2$

$\boxed{V_F = 2.1\ m/s}$

(5) $3m$, 20°, $V_0 = 0$, h_0, $h = 0$

$W_{NC} = E_F - E_0$ so $E_F = E_0$

$W_{NC} = 0$

$\frac{1}{2} m v_F^2 + mgh_F = \frac{1}{2} m v_0^2 + mgh_0$

$h_F = 0$, $v_0 = 0$

$h_0 = 3 - 3\cos 20^\circ$ $\quad$ $\frac{1}{2} m v_F^2 = mgh_0$ $\quad$ $v_F = \sqrt{2gh_0}$

$h_0 = .181\ m$ so $v_F = \sqrt{2 \cdot 9.8 \cdot .181}$

$\boxed{V_F = 1.88\ m/s}$